Worksheet 7 Solutions

Wednesday, March 3, 2021 9:48 AM

1. it is easy to see without computations that the splines need to be lines (uniqueness of interpolating polynomials), but it is good to compute these by hand.

Let the first polynomial be $P_1(x) = a_1x + a_2x^2 + a_3x^3$ and the second $P_2(x) = 1 + b_1(x-1) + b_2(x-1)^2 + b_3(x-1)^3$

(I already used the fact that $P_1(0) = 0$ and $P_1(1) = 1$

agreement on the other data points yields:
$a_1 + a_2 + a_3 = 1$
$1 + b_1 + b_2 + b_3 = 2$

the second derivative vanishing on the boundary gives:
$2a_2 = 0$
$2b_2 + 12b_3 = 0$

finally agreement of first derivatives on interior nodes:
$a_1 + 2a_2 + 3a_3 = b_1$
$2a_2 + 6a_3 = 2b_2$

now we can solve this any way we want, I choose to use linear algebra, this is:

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 12 \\ 1 & 2 & 3 & -1 & 0 & 0 \\ 0 & 2 & 6 & 0 & -2 & 0 \end{pmatrix} \begin{pmatrix} a_1 \\ \mathrm{a}_2 \\ \mathrm{a}_3 \\ \mathrm{b}_1 \\ \mathrm{b}_2 \\ \mathrm{b}_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

we can solve this to get $a_1 = b_1 = 1$ and the rest zero

2. we just modify the second and third row by $a_1 = 1$ and $b_1 + 2b_2 + 3b_3 = 1$, this gives us the same solution.

3. if the data lies on a straight line, then all the cubic polynomials will just be lines for natural splines. For clamped this may not be the case if the prescribed derivative is something different from the function, in that case will have something slightly off.

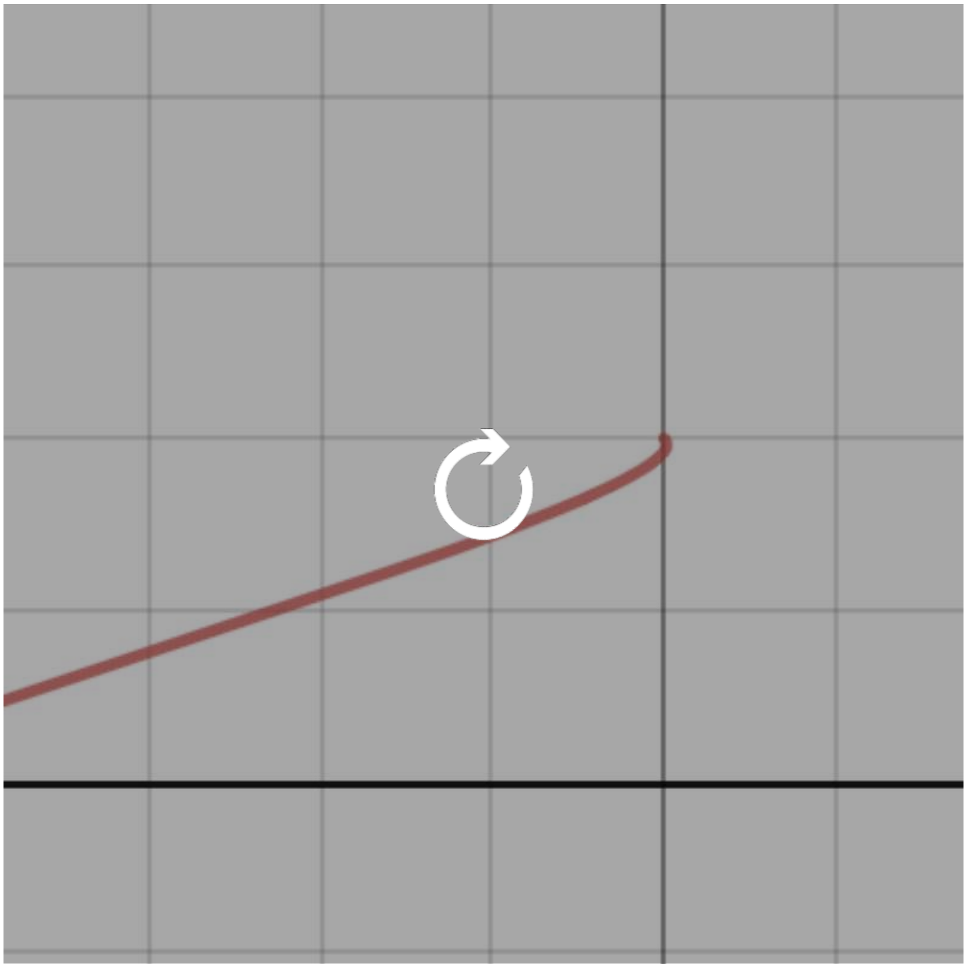
4. (a) we have $x(0) = 0, x(1) = 5, x'(0) = 1, x'(1) = -1$. This can be solved by a system of equatinos or using equation 3.23 on page 164.

we get $x(t) = t + (15-1)t^2 + (-10)t^3 = t + 14t^2 - 10t^3$

similarly, $y(0) = 0, y(1) = 2, y'(0) = 1, y'(1) = 1$

and $y(t) = t + (6-3)t^2 + (-4+2)t^3 = t + 3t^2 - 2t^3$

graph:
parametric graphing



x	f	Forward	backward		Cos		Actual error Forward	Actual Error Backward
0.5	0.4794	0.852			0.877583		0.025583	
0.6	0.5646	0.796	0.852		0.825336		0.029336	0.026664
0.7	0.6442		0.796		0.764842			0.031158

5.

the error bound from the formula is $\frac{h}{2}f''(\xi)$ this is bounded by $\frac{h}{2}$ (since $f'' = -\sin(x)$ is bounded by 1 in absolute value)

now $\frac{h}{2} = \frac{1}{20} = .05$